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Damage detection in initially nonlinear systems

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ABSTRACT

The primary goal of Structural Health Monitoring (SHM) is to detect structural anomalies before they reach a critical level. Because of the potential life-safety and economic benefits, SHM has been widely studied over the past two decades. In recent years there has been an effort to provide solid mathematical and physical underpinnings for these methods; however, most focus on systems that behave linearly in their undamaged state—a condition that often does not hold in complex "real-world" systems and systems for which monitoring begins mid-lifecycle. In this work, we highlight the inadequacy of linear-based methodology in handling initially nonlinear systems. We then show how the recently developed autoregressive support vector machine (AR-SVM) approach to time-series modeling can be used for detecting damage in a system that exhibits initially nonlinear response. This process is applied to data acquired from a structure with induced nonlinearity tested in a laboratory environment.

1. Introduction

With the ubiquitous need to maximize a structure's life while ensuring the health and safety of individuals using the structure, Structural Health Monitoring (SHM) has become a fervently studied topic over the past 20 years. The goal of SHM is to identify damage before it reaches a critical state and at the same time not give false indications of damage. As such, this field has moved rapidly from *ad hoc* and heuristic approaches for damage detection to methodological development based on the concepts of statistical pattern recognition [1] to the point where fundamental axioms for SHM have been proposed [2]. As an example of a system where SHM is important, consider the August, 2007 collapse of the I-35W Mississippi River Bridge. With a robust SHM system in place the structural deficiencies may have been identified and maintenance activities could have been prescribed to prevent the multi-million dollar bridge replacement cost as well as the litigation costs associated with this disaster. At the very least, the bridge might have been decommissioned to prevent the 13 lives lost. This example is just one that highlights the importance of detecting damage at an early stage, prior to irreversible consequences. However, for most applications SHM systems, which include sensing hardware coupled with data interrogation software, have yet to be developed that can be relied upon for such early warnings. As such, despite the extensive literature summarizing SHM research over the last 20 years [3,4] there is still a need for further development and field verification of SHM systems that can provide early indications of potentially dangerous changes in a structure. This capability will allow for maintenance to prevent abrupt failures and extend the structure's life in a cost-effective manner.

To achieve this goal for SHM systems, it is the authors' opinion that a statistical pattern recognition paradigm must be employed in the development of an SHM system. This paradigm consists of a four-step process that includes Operational

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Evaluation, Data Acquisition, Feature Extraction and Feature Classification. Inherent in the last three steps of this paradigm is the need for data normalization, cleansing, and compression, which are implemented with either hardware or software. A more detailed discussion of this statistical pattern recognition paradigm for SHM can be found in [5]. This paper focuses on the feature extraction and classification portions of this paradigm.

Many existing SHM feature extraction methodologies are based on fitting linear models (e.g. a modal model) to measured system response data before and after damage. Changes in the parameters of these models are then used as indicators of damage. This physics-based modeling approach has been extended to data-based time-series models where in addition to model parameters, residual errors between measured and predicted responses are used as damage-sensitive features. More recently, researchers have studied SHM approaches that are based on the principle that an undamaged structure that behaves in an initially linear manner will, under the presence of damage, exhibit nonlinear response [6,7]. The formation of cracks that open and close under load is one such example of damage that would induce nonlinearity. Methods are proposed in [6,7] to take advantage of this transition from linear to nonlinear response in the damage-detection process. However, many real-world structures, particularly those with numerous and complex joints and interfaces, will behave nonlinearly even in their undamaged state. In addition, some situations involve the monitoring of systems that are damaged to begin with. The goal is then to ensure the damage does not increase beyond some critical threshold. This situation may arise, for instance, when the SHM monitoring system is deployed mid-lifecycle. These systems may also demonstrate nonlinear response characteristics in their initial "undamaged" state and hence most existing SHM techniques would be inadequate.

In this paper we begin by focusing on existing SHM methodology, in particular the inability of existing methods to handle initially nonlinear systems. To this end, Section 2 discusses several common techniques for SHM, demonstrating their limitations when applied to initially nonlinear systems. Section 3 focuses on the recent autoregressive support vector machine [8], which by design is well-suited to modeling nonlinear systems. As discussed in [8], support vector machines have been reported many times in the recent SHM literature where they are used as a classification algorithm. However, the study summarized in [8] is, to the authors' knowledge, the first time SVMs have been applied to time-series regression modeling in the SHM context. The study reported herein extends this methodology to data from initially nonlinear systems where the ability to identify damage even between two nonlinear states is demonstrated. The process is demonstrated on a laboratory structure where different nonlinearities can be introduced at various locations. Lastly, Section 4 concludes the work and discusses avenues for further research.

2. Existing SHM feature extraction techniques applied to initially nonlinear systems

With the majority of existing SHM techniques focusing on initially linear systems, we seek to explore how these methods fare when systems behave nonlinearly in their initial state. These techniques primarily seek to fit a predictive model, either physics- or data-based, to the undamaged system. As the system remains in the linear undamaged state these models should continue to accurately predict the system's response. However, at the occurrence of damage, the underlying process generating the data has changed, and hence the model should no longer accurately predict the system's response. Thus one can monitor the model's parameters or its predictive errors as the damage-sensitive feature of interest. An alternate approach is to identify features that directly compare the measured response waveforms or spectra of these waveforms. The extensive number of SHM techniques discussed in the literature precludes a comprehensive summary of each method's ability to handle initially nonlinear systems. Instead, we discuss a few methods reported in the literature to highlight the difficulties that may be encountered when one deals with an undamaged system that exhibits nonlinear response characteristics. This discussion is prefaced with the acknowledgement that the response characteristics exhibited by a structure will depend on the specific type of nonlinearity present and various characteristics may be observed with the same structure if different types of nonlinearities are present.

2.1. Damage-sensitive features derived from model parameters and predictive errors

One data-based technique that has been reported extensively in the SHM literature makes use of traditional autoregressive (AR) models to extract damage-sensitive features. These models are based on a linear fit to the raw time-series sensor output data (at time t with sensor k) of the form

$$x_t^k = \sum_{j=1}^{p} w_j^k \cdot x_{t-j}^k + \varepsilon_t^k. \tag{1}$$

Or stated another way, these models predict the current measured time point based on a linear combination of the p previous measured time points weighted by the parameters w. Here ε_t^k represents the error between the prediction and the actual measured value of the time series at time t. Once the model has been fit to data known to be undamaged, it is then tested on new, potentially damaged data in two manners. First, a similar p-order model can be fit to new data and changes in the parameters, w, between the two models can be used to form a p-dimensional feature vector that is then used to indicate damage. Alternatively, the model trained on the undamaged data can be used to predict the new response data and changes in the properties of the residual error vector can then be used as a damage-sensitive feature. Both these approaches are based on the assumption that if the system continues in its undamaged state, the model (either its parameters or the residual errors) should not change significantly. However, changes in the process generating the data (such as damage) will affect either the model parameters or the residual errors associated with a particular model's prediction. By repeating the model fitting process or the time series prediction over time, either the model parameters or the residuals can then be monitored for significant changes using a variety of statistical procedures such as control charts [9,10]. Statistically significant increases in the variability of these features would then be considered indicative of damage.

As previously stated, each point in the time series is modeled as a linear combination of the previous p measured points in the signal. Thus in order to ensure appropriate fit, and hence the ability of the model to accurately predict other measured responses generated by this system, the response signal must be generated by a linear or near linear process, in which case the AR model gives the best linear approximation to that process. In addition, because of the constrained nature of the AR model, the fit can be poor in signals exhibiting non-stationarity (such as those created by transient hammer-impacts) and lengthy (or no) periodic behavior. Thus when the underlying process is nonlinear in its initial state, and the form of the measured response signal is either unknown or complicated, the linear AR models are not a suitable choice. This difficulty is illustrated with the following numerical example. Consider a signal generated from an undamaged system that has a cubic nonlinearity as described by:

$$m\ddot{y} + c\dot{y} + ky + \alpha y^3 = x(t), \tag{2}$$

where m, c and k are the system's mass, damping and linear stiffness coefficients, respectively, α is a constant associated with the cubic stiffness term, and $x(t)$ is a forcing term. The system response is calculated using a 4th order Runge–Kutta numerical integration scheme [11] with a time step of 0.2 and a maximum time value of $t = 4000$. In addition, the parameters m, c, k and α are set to 1, 0.1, 1, and 1, respectively. The system is subjected to a time-varying harmonic forcing function of the form

$$x(t) = (10 + t/100)\cos(2t). \tag{3}$$

If the system were linear, an increasing amplitude associated with the forcing term would result in a corresponding increase in the response signal, and hence the AR model fit to the normalized response data would fit equally well regardless of input force. However, in a nonlinear system changes in input result in nonlinear changes to the output, and hence the form of the response signal will change. We plot the response for various time intervals in Fig. 1. The first and last intervals shown will be used in the subsequent analysis.

Because the AR model uses a single set of coefficients to model the entire time-series, we expect that it will not adequately capture the system dynamics at all excitation levels when it is fit to data with the nonlinearity present. This point is illustrated in Fig. 2, where the simulated system response to the input described by Eq. (3) and the AR model fit to that data for two different time windows (and hence two different forcing levels) are shown.

This example illustrates the well-known property that the linear AR model does not generalize to predict responses resulting from other input cases when the underlying system is nonlinear. In such cases both of the standard damage-sensitive features associated with these regression models, the model parameters and residual errors, would most likely give false indications of damage. The points being illustrated by this example extends to all the physics-based modeling approaches to damage detection discussed in [3,4] (e.g. modal parameter changes, finite element model updating procedures) that are based on the assumption that the underlying system will exhibit linear response and time-invariant system properties when in its undamaged condition.

2.2. Damage-sensitive features based on waveform comparisons

Alternative damage-detection methods are based on examining the measured response waveforms or spectra of these waveforms from structures in the undamaged condition. Similar waveforms or spectra are obtained from the potentially damaged system and compared to the baseline data in an effort to detect changes in the system producing these data. Many

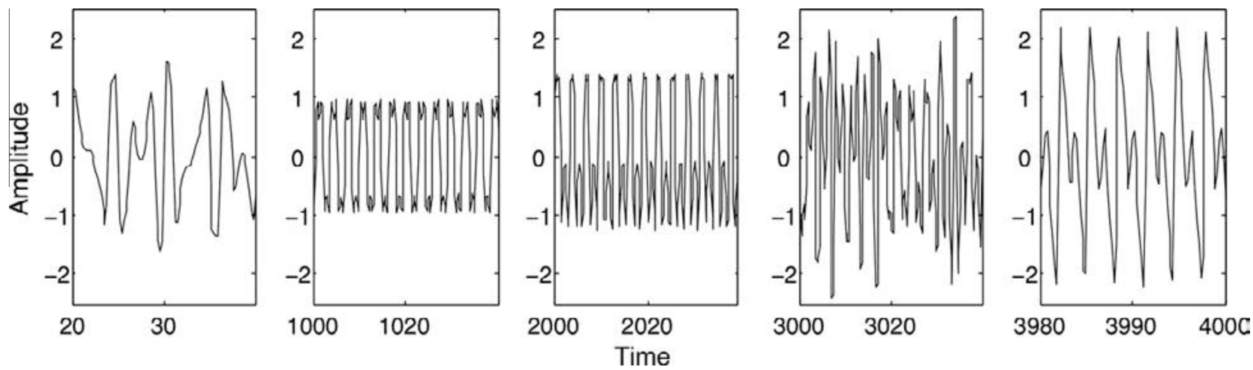

Fig. 1. The response calculated for the system described by Eq. (2) subject to the input described by Eq. (3) for several time intervals. The input amplitude ranges from approximately 10 in the leftmost plot to almost 50 in the rightmost plot.

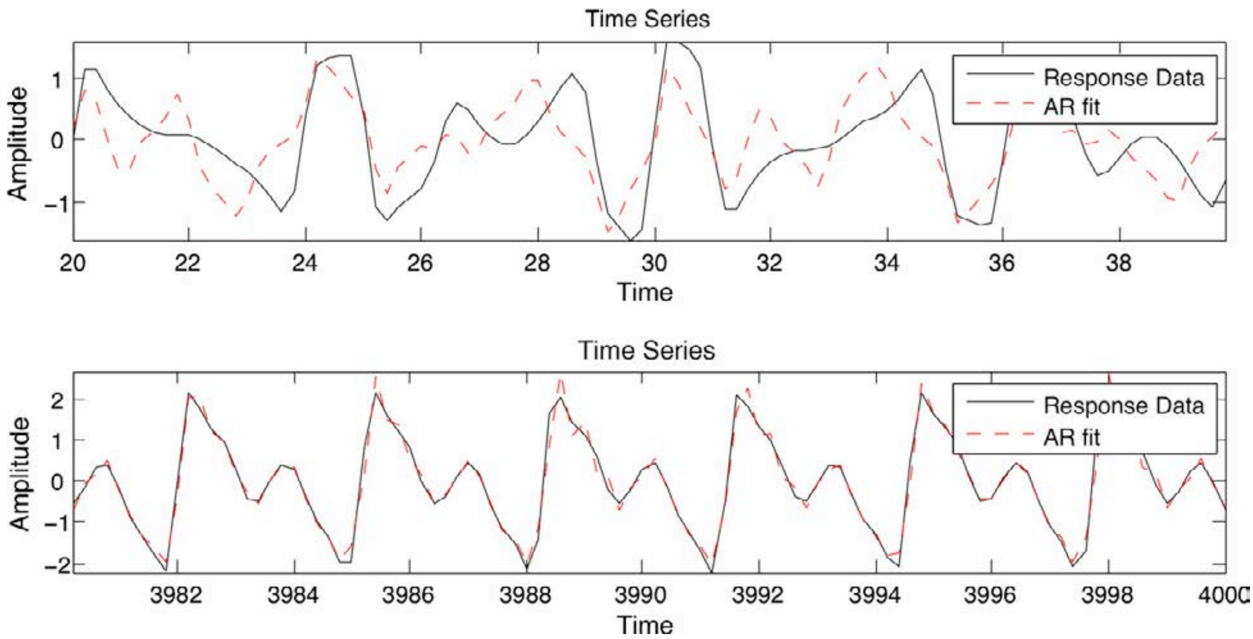

Fig. 2. A comparison of the response calculated for the system described by Eq. (2) subject to the input described by Eq. (3) and the linear AR prediction of that response. The top plot corresponds to the initial portion of the response and the bottom plot corresponds to the terminal portion of the response.

different waveforms or waveform measures can be used to develop damage-sensitive features including statistical moments, temporal moments, linearity checks (exciting the structure at two different levels), reciprocity checks, frequency response functions, coherence functions, probability density functions and harmonic distortions [6]. Many of these approaches are very effective when damage causes the structure to transition from a linear system to a nonlinear system.

The primary difficulty with using waveform comparisons for damage detection in initially nonlinear systems is that the waveforms can vary considerably with different excitation conditions. This variability makes it very difficult to establish thresholds on changes in the waveform that are considered to be indicative of damage. Consider the frequency response function (FRF), which is one such waveform often used in SHM studies. The FRF is defined as the Fourier transform of the response data normalized by the Fourier transform of the excitation. The underlying notion is that the excitation input is filtered by the structural system to create the response output and the FRF defines this filtering process. As an example, if a signal $F_{in}\sin(\omega t)$ is input to a linear single degree of freedom system, the filtering process results in a response at the same frequency, but with different amplitude and with a phase shift, $F_{out}\sin(\omega t + \phi)$. The change in amplitude and the phase shift are a function of the system's mass, stiffness and energy dissipation characteristics. For a linear, time-invariant system the structure's filtering characteristics are independent of the excitation. However, in the presence of nonlinearity, excitation at a given frequency often results in responses at many frequencies. The distribution of energy among these frequencies depends on both the characteristics of the nonlinearity and characteristics of the input force F_{in}, and hence in the nonlinear case the FRF will also be a function of the input energy distribution. As a result, it can be difficult to distinguish effects caused by damage from effects caused by changing inputs, and hence this methodology is not well-suited to detecting damage in initially nonlinear systems.

To illustrate this point, consider a structure discussed in detail in Section 3.2 excited at two different random excitation levels that are a factor of four different in their root-mean-square level. In one case the system is configured so that it responds in a linear manner. In the other case, it is configured such that there is an impact nonlinearity between the top two masses. Fig. 3 shows a plot of the FRF magnitudes for these two cases. For the linear case the FRFs are very consistent from one excitation level to the other as would be expected. However, the FRFs are quite different for the nonlinear case. If the nonlinear case were considered undamaged, it would be very difficult to determine when a new FRF is indicative of damage as opposed to being indicative of a different excitation.

3. An approach to characterizing initially nonlinear systems for SHM

Identifying damage that causes a transition from a linear to nonlinear system is a much simpler task than identifying damage in a system that is initially nonlinear. Despite the difficulties, the detection of damage in initially nonlinear systems is of crucial importance because of the foreseeable large number of "real-world" systems that fall into this category. The question then is what damage-sensitive features may be used in such systems. We have seen that methods designed to detect the transition from linear to nonlinear response are inadequate when the system is initially nonlinear, as are

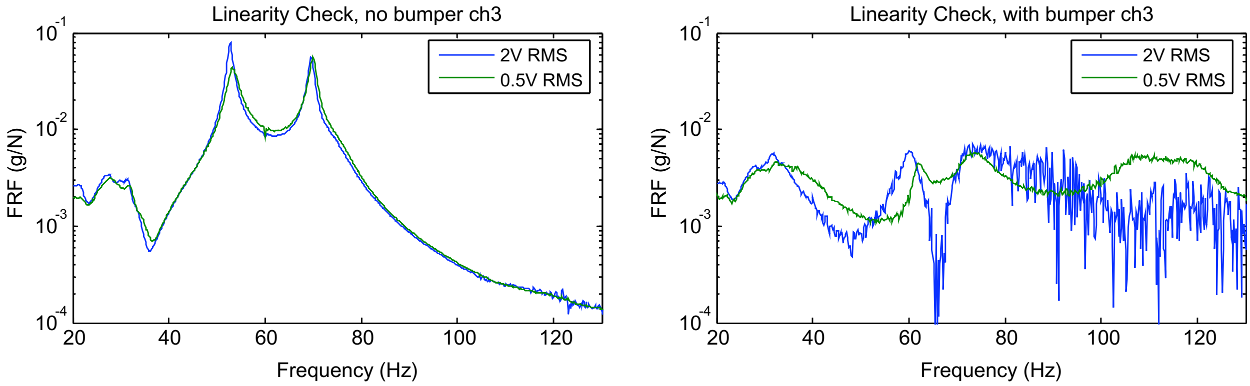

Fig. 3. Frequency response functions corresponding to different excitation levels in a linear system (left) and a nonlinear system (right).

methods based on waveform comparison whose detection features are not well-defined in the presence of nonlinearity. To address these shortcomings, we again focus on the system's time-series data to detect changes in the measured response that are associated with damage. Specifically, we focus on a class of models capable of characterizing highly complex and nonlinear responses. These models, known as autoregressive support vector machines (hereafter AR-SVM), are capable of modeling any nonlinear relationship given appropriate training data. SVMs have been used for SHM before [12–16], however these approaches predominantly focus on one and two class SVMs, which are used for outlier detection and group classification, respectively. In contrast, this study focuses on using SVMs for regression modeling where the damage-sensitive feature will be the residual error between the AR-SVM prediction of the sensor reading and the actual measured time series. Although AR-SVM's robust and precise ability to detect damage has been demonstrated in the linear setting [8], the work described herein constitutes the first effort to use AR-SVMs for damage detection in initially nonlinear systems.

3.1. Autoregressive support vector machines

Following the development first presented in [8,17,18], for the kth sensor and time point t the AR-SVM model has the form

$$\mathbf{x}_t^k = \sum_{j=p+1}^{t_0} \beta_j K(\mathbf{x}_{j-p:j-1}^k, \mathbf{x}_{t-p:t-1}^k) + \varepsilon_t^k, \tag{4}$$

where we have denoted the vector $\{x_j^k, \ldots, x_{t-1}^k\}$ as $\mathbf{x}_{j:t-1}^k$. Here t_0 is the length of the training set on which the model is built, and K is a kernel function, which may by thought of as a measure of distance between two vectors. Although at first glance this model form looks surprisingly like the traditional linear AR model (Eq. (1)), both the development and form are significantly different.

First, assume we have data from a set of measurements without damage, but with nonlinearity present, for time $t = 1, \ldots, t_0$. Next the model order p must be defined. There are many methods for selecting p, such as partial autocorrelation or the Akaike Information Criterion (AIC), which are discussed in more detail in [10]. As with linear AR modeling, we create the training set on which to build our AR-SVM model by using each observation as the dependent variable and the previous p observations as independent variables. Our training samples are thus $\{(\mathbf{x}_{t-p:t-1}^k, x_t^k), t = p+1, \ldots, t_0\}$.

Ideally we would like to find a function $\mathbf{f}$ such that $\mathbf{f}(\mathbf{x}_{t-p:t-1}^k) = x_t^k$ for $t \leqslant t_0$. However, if the form of $\mathbf{f}$ is restricted to linear functions, this restricted form makes perfect fit of the data impossible in most scenarios. As a result, the prediction using $\mathbf{f}$ is allowed to have an error bounded by γ, and then w is found under this constraint. Through the recent advances in penalized regression methods [19,20] it has shown that optimal prediction performance is obtained by minimizing w, which leads us to minimize the Euclidean norm of w subject to the error constraint γ, stated as

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|w\|^2, \\ &\text{subject to} \quad \begin{cases} x_t^k - \langle w, \mathbf{x}_{t-p:t-1}^k \rangle \leqslant \gamma, \\ \langle w, \mathbf{x}_{t-p:t-1}^k \rangle - x_t^k \leqslant \gamma, \end{cases} \end{aligned} \tag{5}$$

where $\langle , \rangle$ denotes the dot product. This development relies on the assumption that a linear model is able to fit the data to within precision γ. However, such a linear model often does not exist, even for moderate settings of γ. As such, we introduce the slack variables ξ_t^+, ξ_t^- to allow for deviations beyond g. The resulting formulation is

$$\begin{aligned}&\text{minimize} \quad \frac{1}{2}\|w\|^2 + C\sum_{t=p+1}^{t_0}(\xi_t^+ + \xi_t^-),\\ &\text{subject to} \quad \begin{cases} x_t^k - \langle w, x_{t-p:t-1}^k\rangle \leqslant \gamma + \xi_t^+, \\ \langle w, x_{t-p:t-1}^k\rangle - x_t^k \leqslant \gamma + \xi_t^-, \end{cases}\end{aligned} \tag{6}$$

where the constant C controls the tradeoff between giving small w and penalizing deviations larger than γ. By framing the above in its Lagrange formulation it can be seen that the optimal $\mathbf{f}$ has the form

$$\mathbf{f}\left(\mathbf{x}_{t-p:t-1}^k\right) = \sum_{j=p+1}^{t_0}\beta_j\left\langle x_{j-p:j-1}^k, x_{t-p:t-1}^k\right\rangle. \tag{7}$$

In this way w may be viewed as a linear combination of the training points $\mathbf{x}_{j-p:j-1}^k$. Note also that in this formation both $\mathbf{f}$ and the corresponding optimization can be described in terms of dot products between the data. Next, the data are transformed from the p-dimensional space to a higher dimension space using a function Φ: $R^p \to F$, and the dot products are computed in the transformed space. Specifically, the mapping allows us to fit linear functions in F which, when converted back to R^p, are nonlinear.

To make use of this transformed space, we replace the dot product term in Eq. (7) with

$$\left\langle \Phi\left(x_{t'-p:t'-1}^k\right), \Phi\left(x_{t-p:t-1}^k\right)\right\rangle. \tag{8}$$

If F is of high dimension, then the above dot product will be extremely expensive to compute. In some cases, however, there is a corresponding *kernel* that is simple to compute. One such family of kernels is the Radial Basis Function (RBF) kernels of the form

$$K(\mathbf{x},\mathbf{y}) = \exp(-\|\mathbf{x}-\mathbf{y}\|^2/(2\sigma^2)), \tag{9}$$

where σ^2 is the kernel variance. This parameter controls fit, with large values leading to smoother functions and small values leading to better fit. Replacing the dot-product in Eq. (8) with the kernel K gives the model defined by Eq. (4).

We now focus on the model's form to identify why it is well-suited for the purpose of modeling nonlinear systems. First, note that the traditional AR model in Eq. (1) fits each point in the time series as a linear combination of the previous p time points. The AR-SVM model instead looks at the previous p points and compares them to every sequence of p points in the training set. Intuitively, if the model notices a sequence of points in the training set very similar to those being tested, it bases its prediction quite heavily on that portion of the training set. This feature allows AR-SVM to perform well when the system exhibits non-stationary response characteristics and when the model order p is too small to observe the entire system response characteristics.

Also, the kernel function K provides the ability to handle nonlinear relationships in the data. Best thought of as a distance measure, kernel functions might take a variety of forms, the most common of which is the radial basis function. While this choice of kernel has been shown to have many good properties, other choices (including the linear dot product) are possible. Through the right choice of kernel function and kernel parameters (such as σ^2 in Eq. (9)), AR-SVM is able to model any nonlinear relationship in the data. While traditional AR models will find the best linear fit to a nonlinear system, the ability to model nonlinear relationships in the data allows AR-SVM to better fit the initially nonlinear data, and hence the predictions from these models will be more sensitive to subsequent changes in the system resulting from damage even when these changes also introduce additional nonlinearities into the system.

It should be noted that RBF neural networks have the same form as Eq. (4). However, fitting these networks requires much more user input such as selecting which β_j are non-zero as well as selecting the corresponding training points. In addition, the fitting of the neural network model is a rather complicated nonlinear optimization process relative to the simple quadratic optimization used in the support vector framework. Although the SVM models are more easily developed, it has been demonstrated [21] that SVMs still more accurately predict the data than the RBF neural networks despite their simplicity.

To demonstrate the ability of AR-SVM to model nonlinear system response, consider the Duffing oscillator previously described in Section 2.1 that was subject to an amplitude varying harmonic excitation defined by Eq. (3). In Fig. 2 it was seen that a linear AR model was unable to predict the system response at different excitation levels. However, Fig. 4, which shows the AR-SVM's prediction of the same two portions of the time series, reveals that AR-SVM is able to capture the system response at the extreme levels of the loading much more accurately than the linear model.

3.2. Experimental example

Most SHM situations require one to consider environmental and operational variability that typically complicates the damage-detection process. However, in order to highlight the AR-SVM method's ability to successfully detect damage in initially nonlinear systems we focus on a controlled laboratory experiment shown schematically in Fig. 5. The experimental structure consists of aluminum plates and columns connected with bolted joints. An electro-dynamic shaker

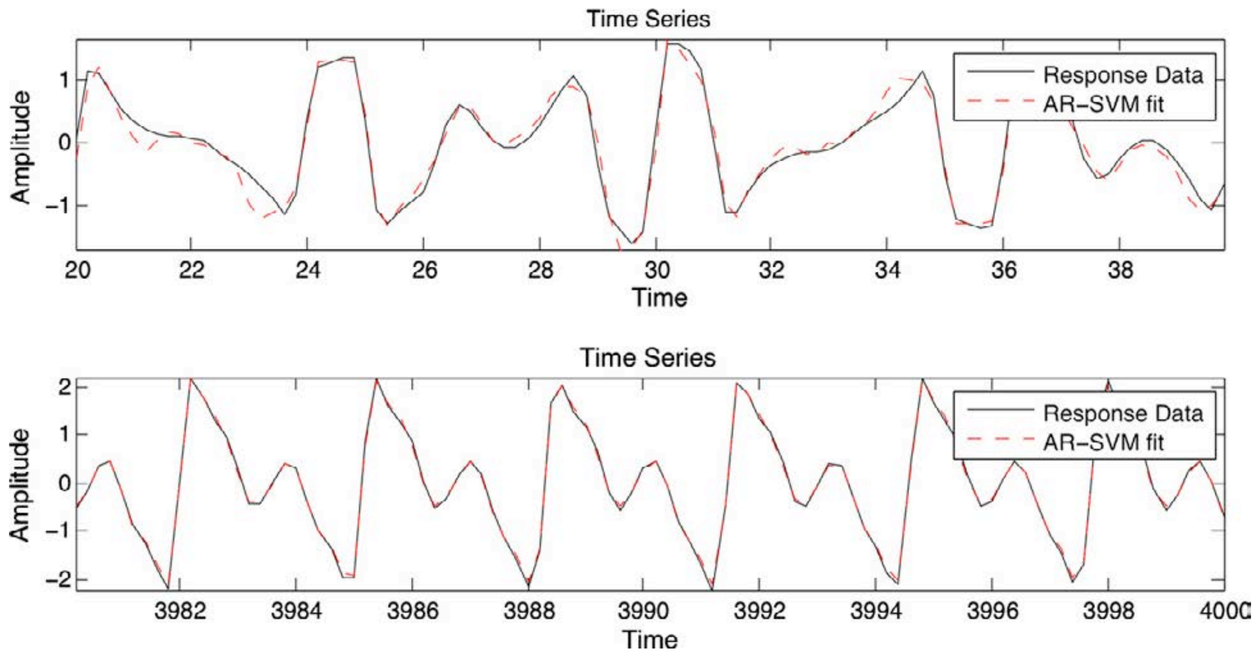

Fig. 4. The AR-SVM prediction of the Duffing oscillator described by Eq. (2) subjected to the input described by Eq. (3).

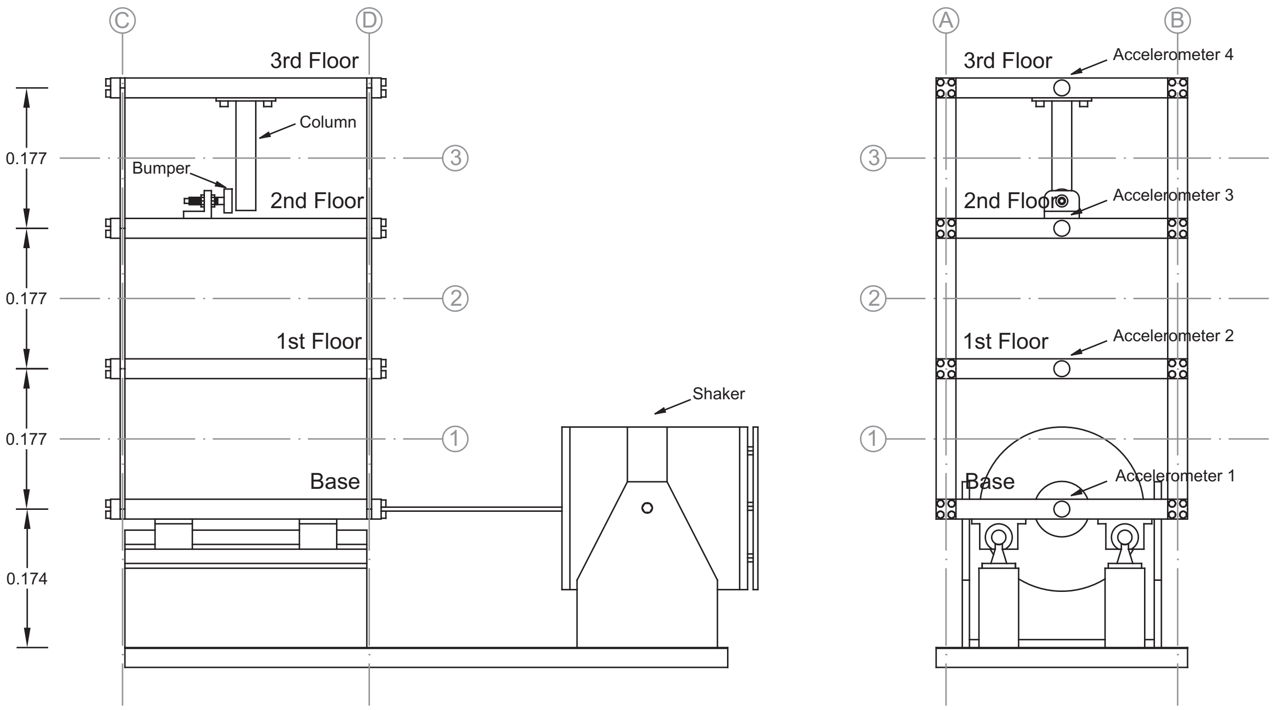

Fig. 5. Diagram of experimental setup.

excites the system through a rigid base constrained to slide horizontally by a system of rails. Each floor contains an accelerometer and is separated from adjacent floors by four corner columns. To simulate nonlinearity in the initial system, a column is suspended from the top floor and a bumper is placed on the second floor. With the bumper gap set at 0.05 mm, the contact of this suspended column with the bumper results in frequent impacts that produce the nonlinear system response that is not considered to be damaged. It is noted that this structure does not represent a scale model of any particular ‘‘real-world” structure, but rather was constructed to exhibit the nonlinear dynamics characteristics that would be observed in real-world structures such as those that might occur if a crack opens and closes under operational conditions.

From this initially nonlinear system we simulate damage by loosening bolts as follows:

Damage 1: loosening bolts connecting column 3BD.
Damage 2: loosening bolts connecting column 2AC.
Damage 3: loosening bolts connecting column 1AD.

Note that these damage cases have similar characteristics to the nonlinearity associated with the undamaged condition in that they both result in repeated impacts when the structure is subjected to the base excitation. The damage cases have the subtle difference in that they also introduce an asymmetry into the structural system stiffness and they correspond to a loss of stiffness from the undamaged condition. The impacting of the bumper associated with the undamaged condition causes an increase in stiffness from the low level response condition when contact is made.

The structure is excited using a band-limited random excitation at 4v RMS in both its undamaged and various damaged conditions. From the resulting accelerometer outputs we seek to identify the presence of damage in each case, as well as any indicators of the damage location, if possible. For a more detailed description of the test structure and the data that were collected, visit http://institute.lanl.gov/ei/software-and-data/.

Sampling at 320 Hz, multiple replicates of response data consisting of 2048 data points were obtained from the structure in both its initial nonlinear undamaged condition and in its damaged condition as described above. Now, we use three of the undamaged nonlinear response replicates (total of 6144 time points) to train the AR-SVM model. Specifically, we normalize these data by subtracting the mean amplitude and then dividing by its standard deviation for each sensor output. Following this data normalization step, we fit the data using the value of $\sigma^2 = 1$ in Eq. (9) and $p = 30$ for Eq. (4). The value of p was determined from the partial autocorrelation plots shown in Fig. 6. These plots display the similarity between observations in the raw undamaged time series as a function of the lag, or time separation, between them. From these plots the smallest model order p should be selected such that the correlation values for lags less than p remain large and for those greater than p remain small.

Now that the AR-SVM models have been trained for each sensor on a length of data known to be damage-free (but with nonlinearities), a test set for each damage case is created consisting of one 2048 time point replicate of the

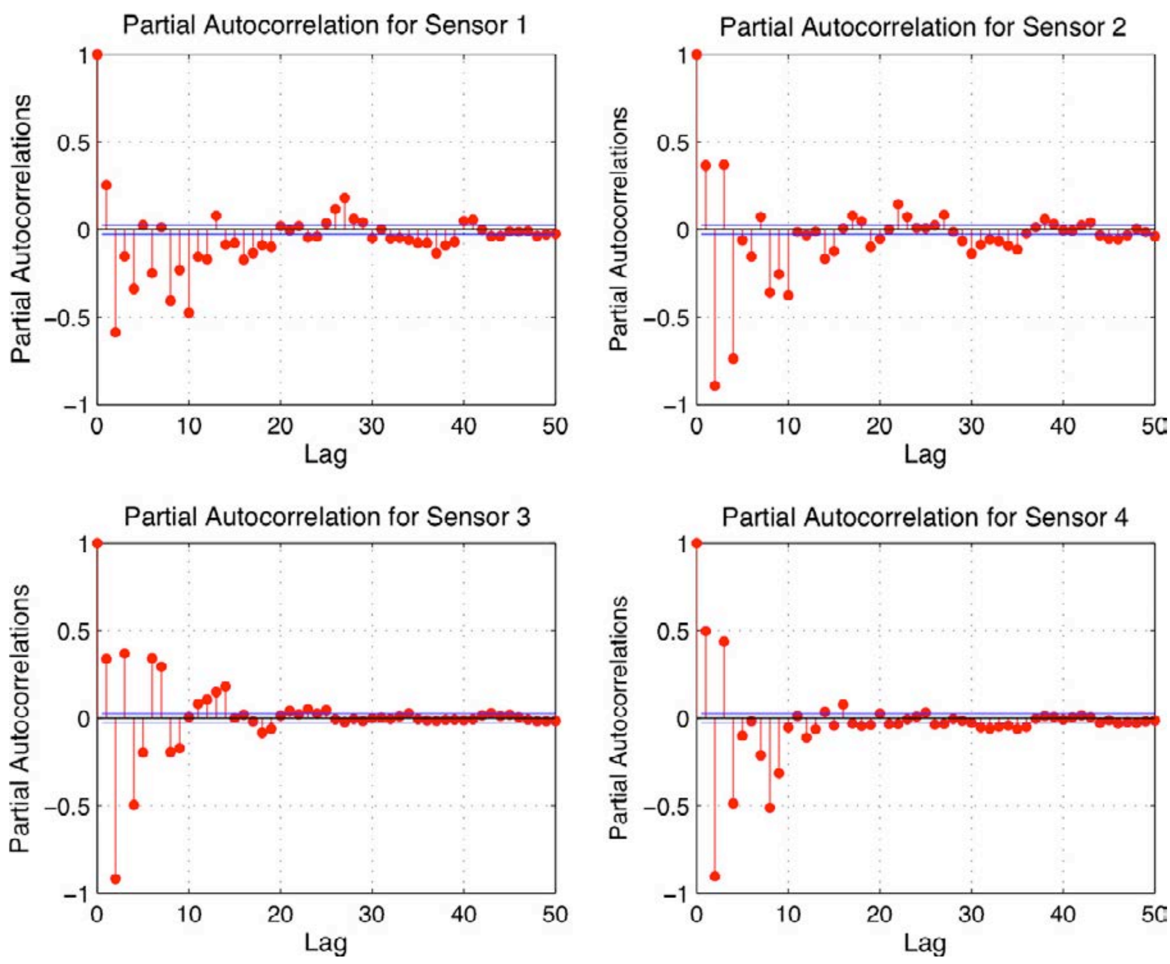

Fig. 6. Partial autocorrelation plots for the four sensors based on the undamaged nonlinear training data.

damaged data concatenated with a similar length record of undamaged data. These concatenated data of length 4096 are our testing data. The goal of the damage-detection process is to recognize a difference between the undamaged nonlinear state (times 0 through 2048) and the damaged state (times 2049 through 4096). The process of analyzing the concatenated data simulates that which might occur with a continuous monitoring system. First, however, the damaged data are normalized using the mean and standard deviation of the training data. Figs. 7–9 plot the residual errors from the model fit to the damaged data for each damage case and sensor.

From these figures it can be seen that AR-SVM is able to differentiate between the undamaged and damaged case. Specifically, the model fits the undamaged portion of the testing set well and fits the damaged portion poorly. Thus the method is quite sensitive to underlying changes in the process, even when the undamaged system responds in a nonlinear manner. Specifically, the associated standard deviations of the residuals from the model fit to the testing data are listed in Table 1. Also listed are the ratios of variances for the undamaged and damaged residuals for each scenario. These ratios will be used to conduct statistical tests that check for changes in AR-SVM model fit to the measured response data and that will be used as the damage indicator.

From these values we see that indeed the residuals for the undamaged portion of the data are much smaller than those in the damaged portion. To be more precise, we may employ an *F*-test to determine if the residuals from the damaged case have significantly larger variance than those from the undamaged case [22]. The *F*-test compares the ratio of variances σ_1^2/σ_2^2 from two normal (or nearly normal) samples, testing the hypothesis that the variances are from the same population (i.e. $H_0 : \sigma_1^2 = \sigma_2^2$). It has been shown, for instance in [8], that the assumption of normal residuals is reasonable for most SHM scenarios. Rejection of this null hypothesis indicates that the sample variances are from different populations as indicated by values of the ratio far from 1. Because of our interest in testing if the variance is increasing as a result of damage, a one-sided test is appropriate. To implement the *F*-test, a significance level must be defined. The significance level is the maximum probability that we have rejected the null hypothesis when, in fact, it should have been accepted. In terms of damage detection this translates into the maximum probability of inferring that damage has occurred when the structure is actually undamaged (also referred to as a false-positive indication of damage). Although in "real-world" damage detection studies the establishment of the significance level will be application dependent and a function of the consequences (e.g. life-safety, economic) of misclassification, in this study a commonly used significance level of 0.05 was chosen.

Because data are available from multiple sensors and, hence, we can perform several *F*-tests, we must account for multiple testing. Therefore, to ensure the *F*-test is conservative, an overall significance of 0.05 will be obtained by testing

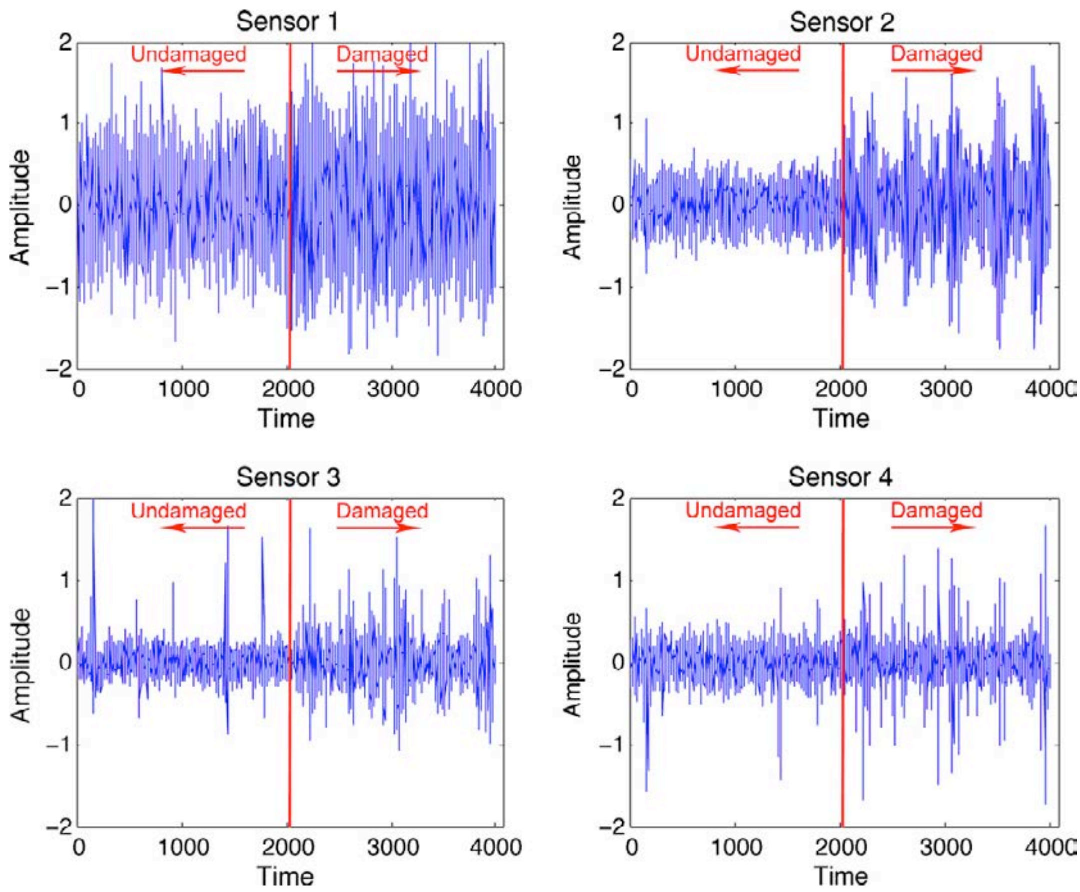

Fig. 7. Residuals from AR-SVM. Damage 1 (loose column connection between floor three and two) at point 2049.

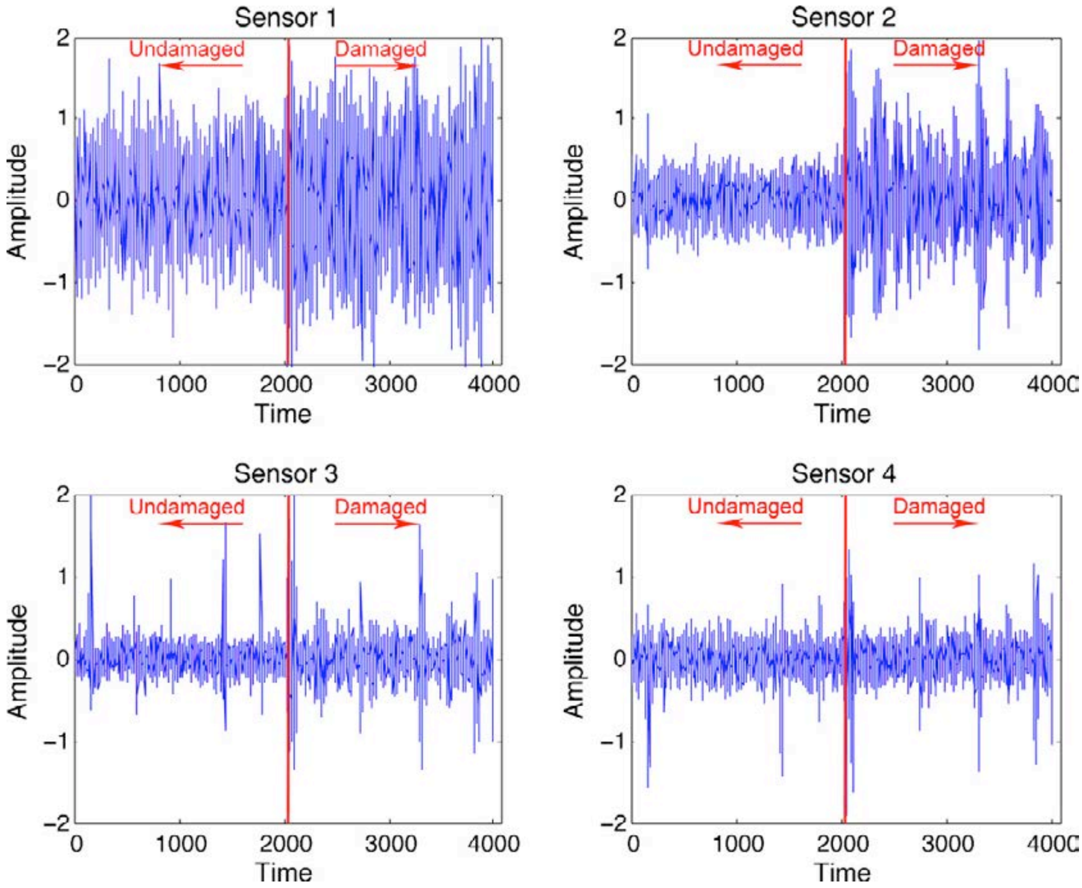

Fig. 8. Residuals from AR-SVM. Damage 2 (loose column connection between floor two and one) at point 2049.

each sensor at significance of 0.01. In fact, such a significance level will result in an overall significance of $1-(1-0.01)^4=0.04$. With 2048 points in both the damaged and undamaged cases, the *F*-test indicates significance at a level of 0.01 if the ratio of variances between undamaged and damaged residuals is less than 0.90 (the undamaged divided by the damaged sample variance). More precisely, a ratio of variances of 0.9 when both samples are of size 2048 corresponds to a *p*-value for the *F*-distribution of 0.009. Similarly, because sensor 2 has a ratio of variances less than 0.36 for all damage scenarios, the *F*-test will on average indicate significance even when we have seen as little as seven damaged time points. This result can be seen from a standard table for an *F*-distribution, which shows a ratio of variances of 0.36 from 2048 and seven samples from the numerator and denominator samples, respectively, corresponds to a *p*-value of 0.01. Hence the method is able to detect changes in the system even at a very early stage. We conclude that AR-SVM is fitting the undamaged data significantly better than the damaged data, and hence the method is able to differentiate between the two conditions.

Note that the results of the significance test do not necessarily indicate the damage location. However, we can determine which sensor (in this case sensor 2) is most sensitive to changes in the system. Sensor 3 is an interesting case, fitting the third damage scenario slightly better than the undamaged case. Because of this sensor's close proximity to the initial nonlinearity and large distance from the damage source, it is likely that the damage had little effect, and the slightly better fit to this case (residual standard deviation of 0.17 versus 0.18) is merely due to rounding error and statistical variation.

4. Conclusions

In this paper we have noted that not all ''real-world'' systems of interest to Structural Health Monitoring experts fit the initially linear profile typically studied in the SHM literature. If there is intent to retrofit SHM systems to old structures as well as new structures with complex joints and interfaces and to have such systems analyze data from widely varying operational and environmental loading conditions, these systems must be able to handle nonlinearity in the initial system state. The discussion first focused on general classes of currently used SHM methods and demonstrated that each is not well-suited to handle systems exhibiting initial nonlinearity. Then the AR-SVM methodology was developed and shown to accurately model nonlinearity associated with the initial undamaged system. This method was then shown to be able to detect changes in the dynamic system response associated with damage that produced another source of nonlinearity with similar characteristics as the nonlinearity associated with the undamaged condition. The authors consider this to be a very challenging

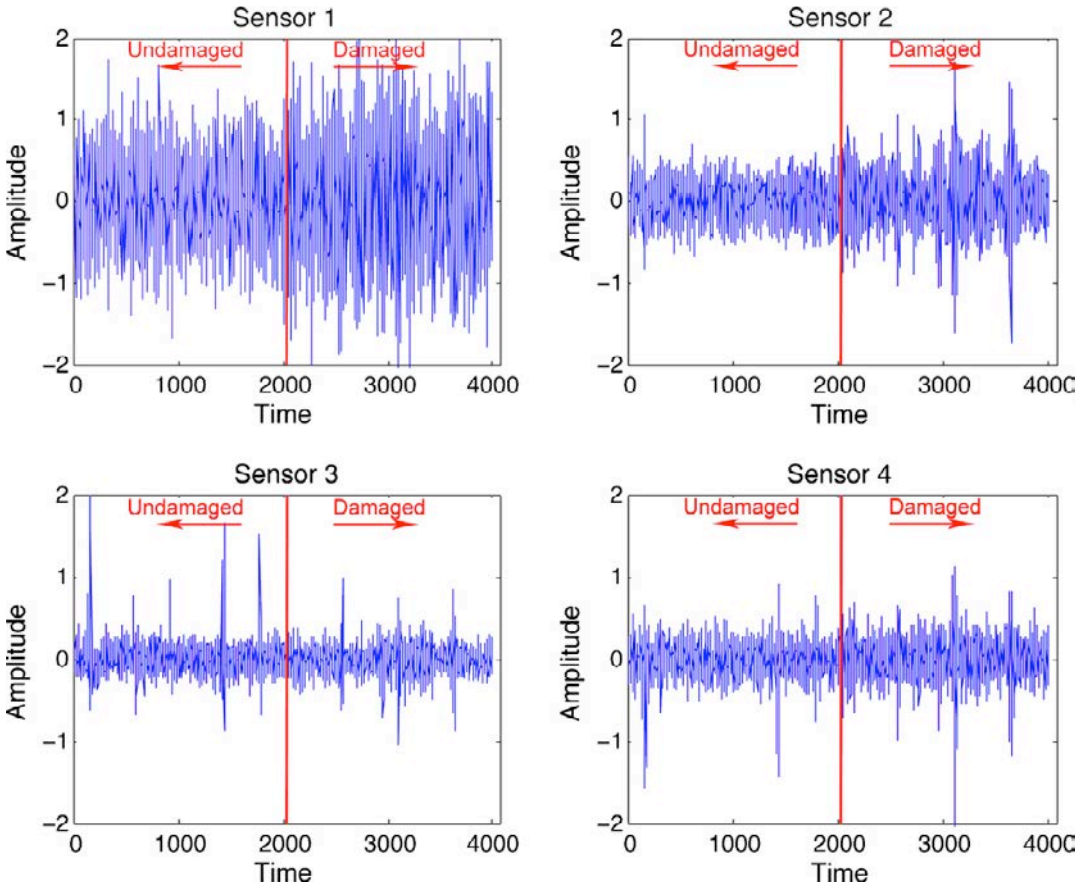

Fig. 9. Residuals from AR-SVM. Damage 3 (loose column connection between floor one and the base) at point 2049.

Table 1
Standard deviations of AR-SVM model residuals.

	Sensor 1	Sensor 2	Sensor 3	Sensor 4
Undamaged[a]	0.42	0.20	0.18	0.18
Damage 1[b]	0.63 (0.44)	0.49 (0.17)	**0.27 (0.45)**	**0.26 (0.48)**
Damage 2[b]	0.69 (0.37)	**0.51 (0.15)**	**0.28 (0.41)**	0.25 (0.52)
Damage 3[b]	**0.66 (0.41)**	**0.33 (0.36)**	0.17 (1.12)	0.24 (0.56)

The ratio of variances for undamaged/damaged is shown in parentheses for each scenario.
[a] Source of initial nonlinearity considered to be part of the undamaged system is located between sensors 3 and 4.
[b] The bold entries are the sensors on the floors between which the column was loosened to simulate damage.

damage detection case that is indicative of challenges that real-world applications pose for SHM processes and that highlights the strengths of the AR-SVM methodology for SHM applications. As with all damage-detection methods, AR-SVM is not immune to the challenges accompanying systems that behave very similarly in both their undamaged and damaged state. Specifically, if damage is present in the training set, this damage will not be detected as such in the testing set. In addition, damage that manifests itself in a form similar to the initial nonlinearities will not be detected. However, it begs repeating that all vibration-based SHM methods suffer similar flaws.

As a next step, work must be done to identify damage in nonlinear systems even in the presence of environmental variability. Because real-world systems are subject to varying loads and other conditions, SHM methods for these systems must be sensitive to damage, yet robust to changes in environment. However, distinguishing between the two can be a difficult problem. This step requires the application of these processes to data from *in situ* structures subject to environmental and operational variability. However, it is often difficult to find such structures that subsequently experience damage on a reasonable time scale as is needed for a proof-of-concept demonstration.

By detailing the need to advance the study of initially nonlinear systems, we hope this work sparks interest and development in this area. Without the development of tools for handling such systems, practitioners may be forced to use alternative linear-based methods that may fail. In general, it is the authors' speculation that such methods will fail through false-negative indications of damage caused by lack of sensitivity to system change that occurs when linear models attempt to model nonlinear system response. Thus, the need for methodology related to nonlinear systems is not only to increase the scope of SHM applications, but also to increase the feasibility and workability of SHM systems in any system potentially exhibiting nonlinearity. With the addition of this work, the AR-SVM method has been shown to be effective in modeling both linear and nonlinear systems, and hence is a good starting point for practitioners dealing with initially nonlinear systems.

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